Comparing the double-cone shape with the base-diameter-axis to a cube rotating around any central-axis using the same equivalent measures, we can quickly get to;

$$I_{diamond} = \frac{3}{10} M_{dc} \left(R_{dc}\right)^2 + \frac{1}{5} M_{dc} \left(h_{dc}\right)^2$$

$$I_{cube} = \frac{1}{6} M_{cube} \left(S_{cube}\right)^2$$

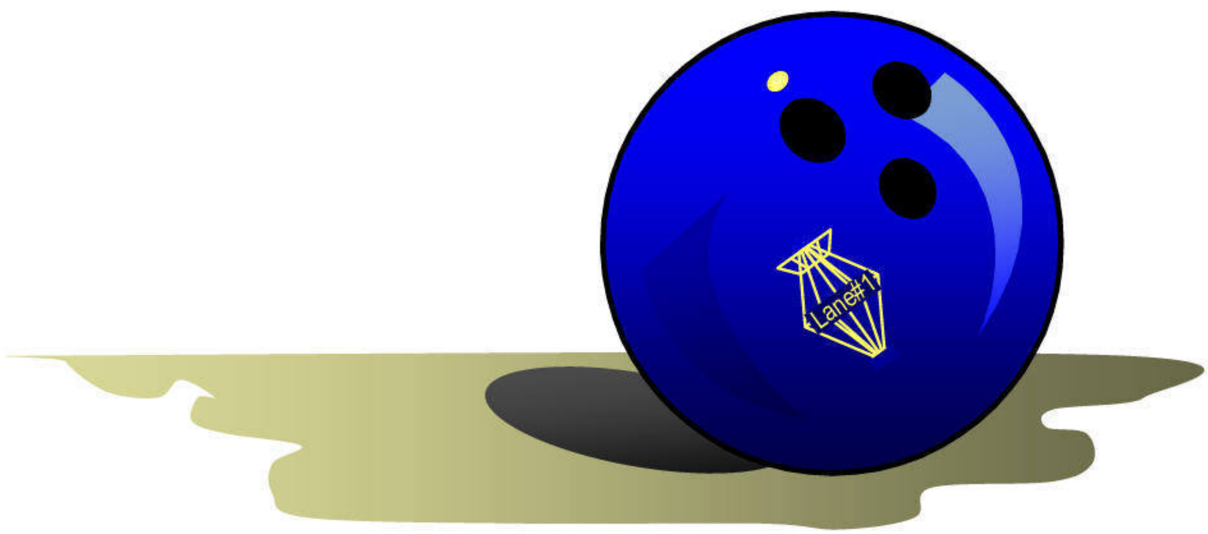
$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} M_{dc}\left(R_{dc}\right)^2 + \frac{1}{5} M_{dc}\left(h_{dc}\right)^2\right)\omega_{diamond}^{\;2}}{\frac{1}{2}\left(\frac{1}{6} M_{cube}\left(S_{cube}\right)^2\right)\omega_{cube}^{\;2}}$$

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} MR^2 + \frac{1}{5} MR^2\right)}{\frac{1}{2}\left(\frac{1}{6} MR^2\right)} = \frac{\frac{1}{2}\left(3/10 + 1/5\right)}{\frac{1}{2}\left(1/6\right)} \cong 3.0$$

the same result as was found earlier when just the moment of inertia was compared.

When comparing the moment of inertia of the core shapes we can't help but to also compare rotational kinetic energy differences the core shapes.

We have discovered that a diamond shaped core will have more rotational kinetic energy than a solid cubical shaped core under nearly equivalent measurement and rotational conditions.



THEORETICAL COMPARISONS: MOMENT OF INERTIA

Rotational Inertia Considerations: Sphere vs. Diamond Core

The list of basic assumptions is an attempt to hold constant many variables in the design of a bowling ball core so that only the three dimensional shape and mass distribution can be directly compared to create meaningful comparisons and will be used to make some predictions on some of the physical parameters and dynamics of a bowling ball. To make comparisons based on the "shape alone" and to determine if there is an advantageous core shape the following assumptions will highlight possible physical differences.

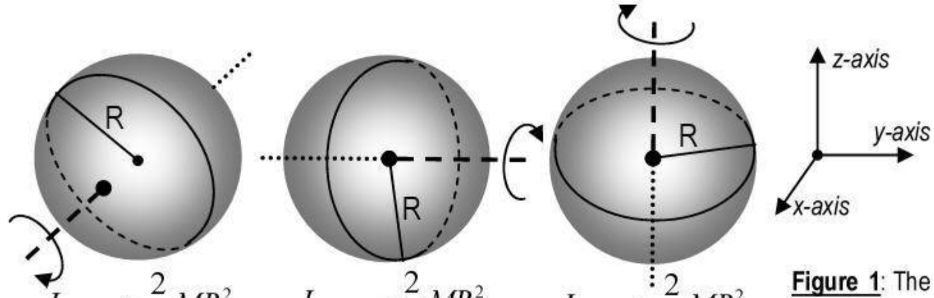
Basic Assumptions for Comparisons:

1. Each core is of similar dimension, constant density, and will have the same mass. (equal size, weight, and smooth mass distribution)
2. Each core is set revolving about is rotational axis to the same rotational velocity. (equal spin speed)
3. Each core rotates about an axial geometric center. (axis through center)

These assumptions of equivalence will highlight differences in core shape designs. These first direct comparisons are core shape to core shape only and *do not* include the rest of the bowling ball. Core shape design dynamics that includes the effect of the filler and coverstock will be attempted in a future research project.

Rotational inertia (moment of inertia) of an object describes how difficult it is to induce an angular rotation of the object about a given axis through that object. Simply, an object of mass resists a change in angular velocity. If a body has a large moment of inertia, then it is difficult to change its angular velocity. If it has a small moment of inertia, it is easier to change its angular velocity. The moment of inertia for any object depends on a number of factors including the object's mass, its shape, and the axis of rotation.

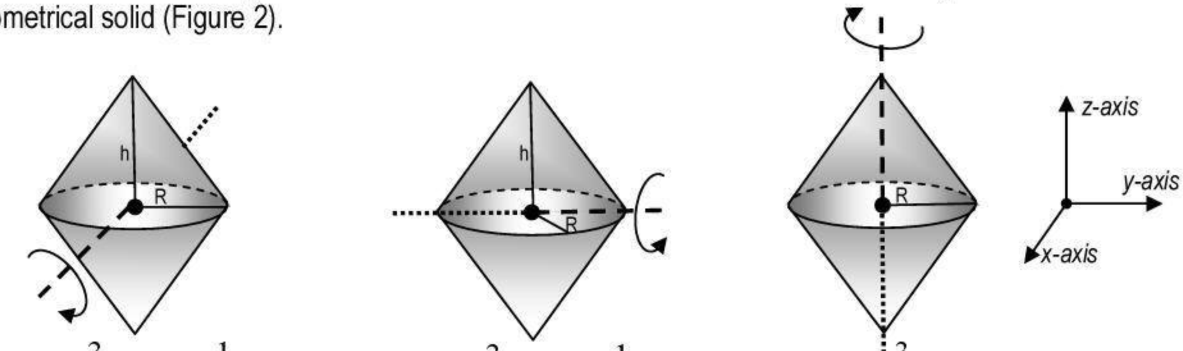
The calculated theoretical moment of inertia for a solid spherical mass (I) was determined to be the same for any given axis through its geometrical center (due to the high degree of symmetry—Figure 1).



$$I_{x-axis} = \frac{2}{5}MR^2 \qquad I_{y-axis} = \frac{2}{5}MR^2 \qquad I_{z-axis} = \frac{2}{5}MR^2$$

Figure 1: The moment of inertia (I) formulae around each principle axis for a solid sphere.

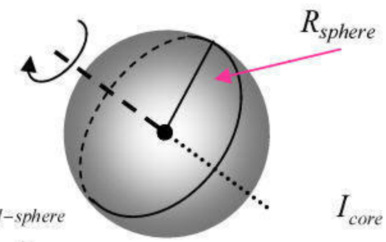
The calculated theoretical moment of inertia for a double solid cone (I) was determined to be the same for two axes through the base of each cone while different for an axis through the vertices of the geometrical solid (Figure 2).



$$I_{x-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2 \qquad I_{y-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2 \qquad I_{z-axis} = \frac{3}{5}MR^2$$

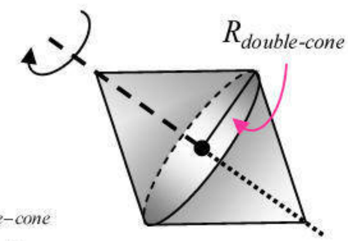
Figure 2: The moment of inertia (I) formulae around each principle axis for a double solid cone (approximate diamond core shape).

Now a comparison: If we compare a "diamond core" (double-right cone) to a spherical core around the following central rotational axes we can see that the rotational inertia will be different for each core since the calculated moment of inertia formula differs due to the different mass distribution about the rotational axis for each geometric solid shape (Figure 3).



$$I_{core} = I_{solid-sphere}$$

$$I_{solid-sphere} = \frac{2}{5}M_{sphere}\left(R_{sphere}\right)^2$$



$$I_{core} = I_{double-cone}$$

$$I_{double-cone} = \frac{3}{5}M_{double-cone}\left(R_{double-cone}\right)^2$$

Figure 3: The comparison of moment of inertia formulae for each core shape.

We can begin our first attempt at comparing the dynamic differences of core shape design by doing a direct comparison of the important quantities that govern the measured and theoretical moment of inertia of these rotating solids. We will employ a simple physical ratio of the rotational inertia formula for each object to highlight any difference core shape may yield.

$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} \approx \frac{\left(\frac{3}{5}M_{double-cone}\left(R_{double-cone}\right)^2\right)}{\left(\frac{2}{5}M_{sphere}\left(R_{sphere}\right)^2\right)}$$

Using ***Assumption #1*** described earlier that each core is of constant density, both cores have the same mass, and each is of similar dimension (where the radius of the spherical core is equal to the radius of double cone), then;

$$M_{double-cone} = M_{sphere} = M$$

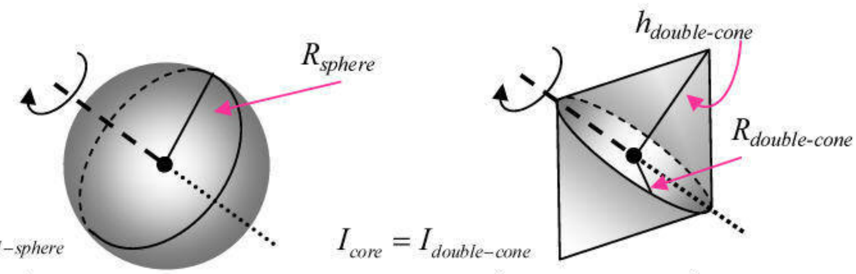
and

$$R_{double-cone} = R_{sphere} = R$$

$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} \approx \frac{\left(\frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2\right)}{\left(\frac{2}{5} M_{sphere} \left(R_{sphere}\right)^2\right)} \cong \frac{\left(\frac{3}{5} MR^2\right)}{\left(\frac{2}{5} MR^2\right)} \cong \frac{3/5}{2/5} \cong 1.5$$

All we have done is compare the moment of inertia of two cores where the only principle difference between them is the mass distribution about the rotational axis. This shows that the shape alone of a core creates different physical results for the rotational analog to mass. Rotational inertia describes an object's resistance to a change in its rotational motion, therefore, this result shows that the double-solid-cone (approximate diamond core) has more rotational inertia. Once the double-cone core is revolving with a given angular velocity (spin speed) it will resist changes to that angular velocity more than the solid sphere core.

Now a different comparison: Now we compare a "diamond core" (double-right cone) to a spherical core around the following rotational axes. (This would have the diamond core rotating end over end— Figure 4.)



$$I_{core} = I_{solid-sphere} \qquad I_{core} = I_{double-cone}$$

$$I_{solid-sphere} = \frac{2}{5} M_{sphere} \left(R_{sphere}\right)^2 \qquad I_{double-cone} = \frac{3}{10} M_{dc} \left(R_{dc}\right)^2 + \frac{1}{5} M_{dc} h_{dc}^{\ 2}$$

Figure 4: The comparison of moment of inertia formulae for each core shape: sphere rotating about its center and double-cone rotating end over end.

Again, using a physical ratio of the rotational inertia formula for each object we get the following mathematical expression;

$$\frac{\text{Diamond Core Ball}}{\text{Sphere Core Ball}} \approx \frac{\left(\frac{3}{10} M_{double-cone} \left(R_{double-cone}\right)^2 + \frac{1}{5} M_{double-cone} \left(h_{double-cone}\right)^2\right)}{\left(\frac{2}{5} M_{sphere} \left(R_{sphere}\right)^2\right)}$$

This expression may look daunting, but we can draw some conclusions if we are smart in making the comparison. Using ***Assumption #1*** with cores of constant density, cores that are of equal mass, and each is of similar dimension (where the radius of the spherical core is equal to the radius and height of the double cone), then;

$$M_{double-cone} = M_{sphere} = M$$

and

$$R_{double-cone} = R_{sphere} = h_{double-cone} = R$$

Comparing with ***assumption #1*** and the cores rotating about these axes, we get the following result;

$$\frac{\text{Diamond Core Ball}}{\text{Sphere Core Ball}} \approx \frac{\left(\frac{3}{10}M(R)^2 + \frac{1}{5}MR^2\right)}{\left(\frac{2}{5}MR^2\right)} \cong \frac{3/10 + 1/5}{2/5} \cong 1.25$$

This again shows that the shape alone of a core creates different physical results. Even about the different axis once the double-cone core is revolving with a given angular velocity (spin speed) it will resist changes to that angular velocity more than the solid sphere core. If we picked a rotational axis somewhere between the two extremes of the double-cone core shape (base-axis or vertex-to-vertex axis) the moment of inertia (ratios) values will be somewhere between the two results (1.25-1.50) that have been obtained. Therefore, it is safe to say that the solid double-cone core shape will have more rotational inertia than the simple spherical solid shape of the same constant density, mass, and relative size.

Theoretical Comparisons

Rotational Energy Considerations: Sphere vs. Diamond Core

So, besides, the idea that shape will influence the dynamics of a core what else can we determine with these moment of inertia formulae ratios? Using ***Assumption #2*** that each core is set revolving about is rotational axis to the same rotational velocity we can delve into the realm of the “energy” content of a rotating core of a specific shape and highlight the differences when we attempt energy considerations.

Recall Rotational Kinetic Energy is given by the equation:

$$K.E._{rotational} = \frac{1}{2}I\omega^2$$

Kinetic energy of rotation depends not only on angular velocity (ω) but also on the moment of inertia (I) of the spinning object (in this case—core). The rotational inertia of a core can be varied with different designed mass configurations (different core shapes).

So a we can make simple ratios to show the difference in the rotational kinetic energy that a spherical core will contain versus the amount of double-cone core shape.

Comparing the double-cone shape with the vertex to vertex axis to a solid sphere rotating around any principle axis.

$$\frac{\text{Diamond Core } KE_{rotational}}{\text{Solid Sphere Core } KE_{rotational}} = \frac{\frac{1}{2}\left(I_{diamond}\right)\omega_{diamond}^{\;2}}{\frac{1}{2}\left(I_{sphere}\right)\omega_{sphere}^{\;2}}$$

$$I_{diamond} = \frac{3}{5}M_{diamond}R_{diamond}^2$$

$$I_{sphere} = \frac{2}{5}M_{sphere}R_{sphere}^2$$

$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} = \frac{\frac{1}{2}\left(\frac{3}{5}M_{diamond}R_{diamond}^2\right)\omega_{diamond}^{\;2}}{\frac{1}{2}\left(\frac{2}{5}M_{sphere}R_{sphere}^2\right)\omega_{sphere}^{\;2}}$$

Now, reminding ourselves that we are investigating the importance of the mass distribution of a bowling ball core shape, we can clearly highlight the rotational energy content differences of the two. With both objects spinning with the same rotational velocity (ω), both with the same overall mass (M), and, finally, both cores assumed to be the same size (R) the preceding mathematical ration reduces to a result we have already seen.

$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} = \frac{\frac{1}{2}\left(\frac{3}{5}MR^2\right)\omega^2}{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2} = \frac{\frac{1}{2}\left(\frac{3}{5}\right)}{\frac{1}{2}\left(\frac{2}{5}\right)} = \frac{3}{2} \cong 1.5$$

Comparing the double-cone shape with the base-axis to a solid sphere rotating around any principle axis using the same equivalent measures, we can quickly get to;

$$I_{diamond} = \frac{3}{10}M_{dc}\left(R_{dc}\right)^2 + \frac{1}{5}M_{dc}\left(h_{dc}\right)^2$$

$$I_{sphere} = \frac{2}{5}M_{sphere}R_{sphere}^2$$

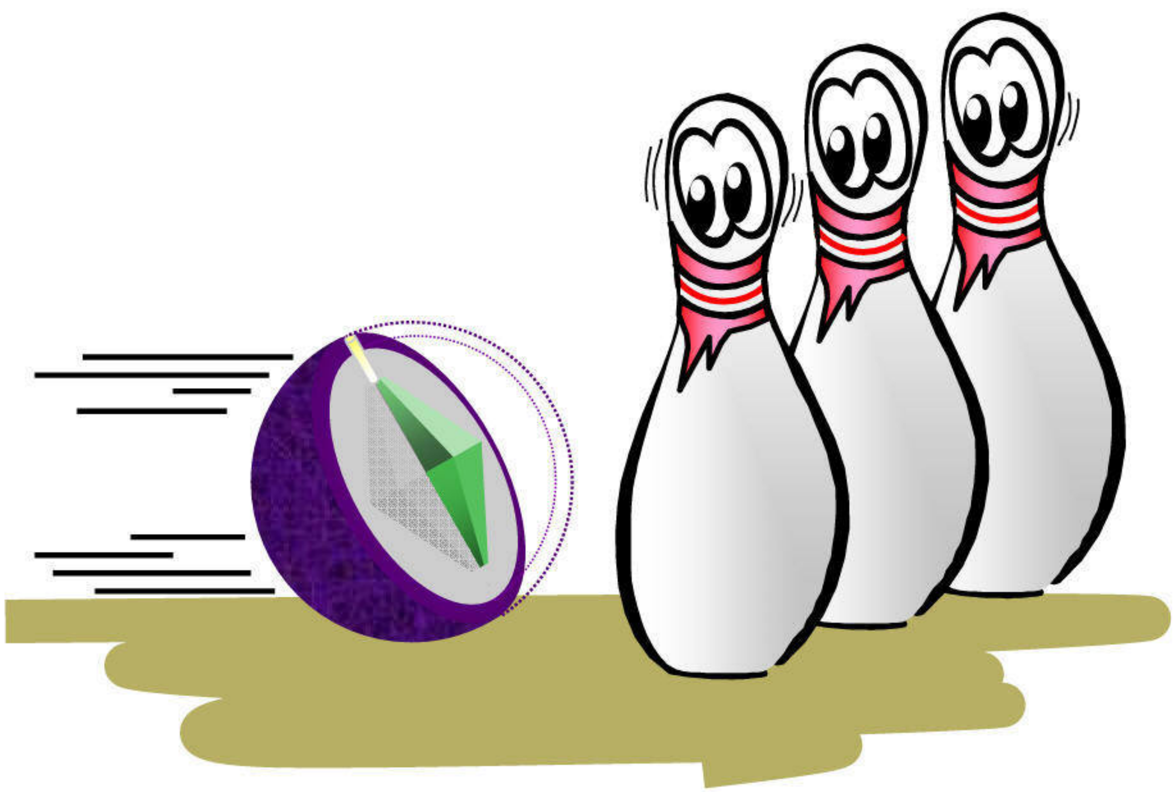
$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} = \frac{\frac{1}{2}\left(\frac{3}{10}M_{dc}\left(R_{dc}\right)^2 + \frac{1}{5}M_{dc}\left(h_{dc}\right)^2\right)\omega_{diamond}^{\;2}}{\frac{1}{2}\left(\frac{2}{5}M_{sphere}R_{sphere}^2\right)\omega_{sphere}^{\;2}}$$

$$\frac{\text{Diamond Core}}{\text{Solid Sphere Core}} = \frac{\frac{1}{2}\left(\frac{3}{10}M\left(R\right)^2+\frac{1}{5}MR^2\right)}{\frac{1}{2}\left(\frac{2}{5}MR^2\right)} = \frac{\frac{1}{2}\left(\frac{3}{10}+\frac{1}{5}\right)}{\frac{1}{2}\left(\frac{2}{5}\right)} \cong 1.25$$

the same result as was found earlier when just the moment of inertia was compared.

When comparing the moment of inertia of the core shapes we are also comparing the possible rotational kinetic energy differences between the two core shapes as long as the cores have similar density, mass, dimension, and equivalent rotational speed.

We have discovered that a diamond shaped core will have more rotational kinetic energy than a solid spherical shaped core under equivalent measurement and rotational conditions.



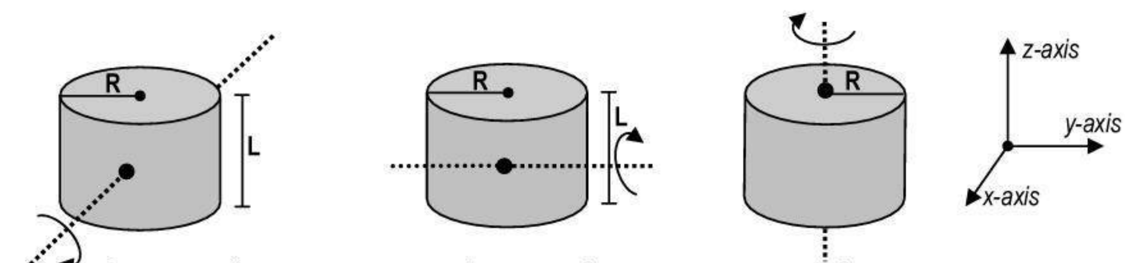
THEORETICAL COMPARISONS: MOMENT OF INERTIA

Rotational Inertia Considerations: Cylinder vs. Diamond Core

Basic Assumptions for Comparisons:

1. Each core is of similar dimension, constant density, and will have the same mass. (equal size, weight, and smooth mass distribution)
2. Each core is set revolving about is rotational axis to the same rotational velocity. (equal spin speed)
3. Each core rotates about an axial geometric center. (axis through center)

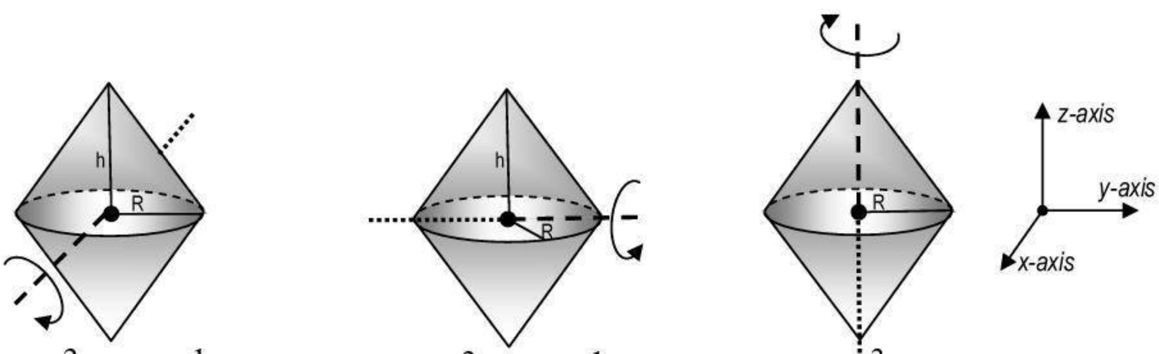
The calculated theoretical moment of inertia for a solid cylinder mass (I) was determined to be the same for any given axis through its geometrical center parallel to the cylindrical base (due to the high degree of symmetry—Figure 1), but different with a central rotational axis parallel to its side .



$I_{x-axis} = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ $\quad$ $I_{y-axis} = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ $\quad$ $I_{x-axis} = \frac{1}{2}MR^2$

Figure 1: The moment of inertia (I) formulae around each principle axis for a solid cylinder.

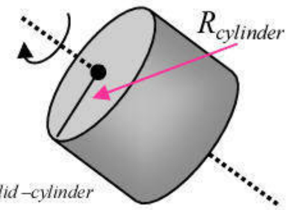
The calculated theoretical moment of inertia for a double solid cone (I) was determined to be the same for the axes through the base of each cone while different for an axis through the vertices of the geometrical solid (Figure 2).



$I_{x-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2$ $\quad$ $I_{y-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2$ $\quad$ $I_{z-axis} = \frac{3}{5}MR^2$

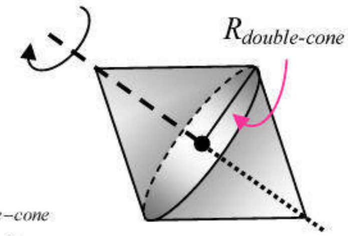
Figure 2: The moment of inertia (I) formulae around each principle axis for a double solid cone (approximate diamond core shape).

<u>Now a comparison</u>: If we compare a "diamond core" (double-right cone) to a cylindrical core around the following central rotational axes we can see that the rotational inertia will be different for each core since the calculated moment of inertia formula differs due to the different mass distribution about the rotational axis for each geometric solid shape (Figure 3).



$$I_{core} = I_{solid-cylinder}$$

$$I_{solid-cylinder} = \frac{1}{2} M_{cylinder} \left(R_{cylinder}\right)^2$$



$$I_{core} = I_{double-cone}$$

$$I_{double-cone} = \frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2$$

<u>Figure 3</u>: The comparison of moment of inertia formulae for each core shape.

We will employ a simple physical ratio of the rotational inertia formula for each object to highlight any difference core shape may yield. Using ***Assumption #1*** described earlier that each core is of constant density, both <u>cores</u> have the same mass, and each is of similar dimension (where the radius of the cylinder core is equal to the radius of double cone), then;

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} \approx \frac{\left(\frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2\right)}{\left(\frac{1}{2} M_{cylinder} \left(R_{cylinder}\right)^2\right)}$$

$$M_{double-cone} = M_{cylinder} = M$$

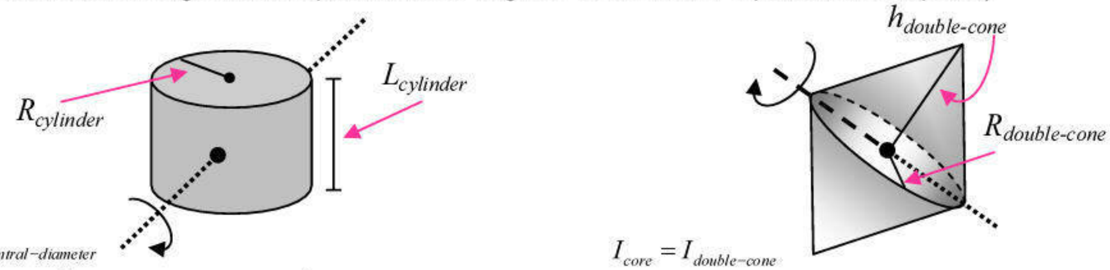
$$\text{and}$$

$$R_{double-cone} = R_{cylinder} = R$$

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} \approx \frac{\left(\frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2\right)}{\left(\frac{1}{2} M_{cylinder} \left(R_{cylinder}\right)^2\right)} \cong \frac{\left(\frac{3}{5} MR^2\right)}{\left(\frac{1}{2} MR^2\right)} \cong \frac{3/5}{1/2} \cong 1.2$$

All we have done is compare the moment of inertia of two cores where the only principle difference between them is the mass distribution about the rotational axis. Once the double-cone core is revolving with a given angular velocity (spin speed) it will resist changes to that angular velocity more than the solid cylinder core.

Now a different comparison: Now we compare a "diamond core" (double-right cone) to a cylindrical core around the following rotational axes. (This would have the diamond core rotating end over end which occurs in a stacked drilling when the pin axis is 90 degrees to the bowler's preferred axis point.)



$I_{core} = I_{central-diameter}$

$I_{central-diameter} = \frac{1}{4}M_{cylinder}\left(R_{cylinder}\right)^2 + \frac{1}{12}M_{cylinder}\left(L_{cylinder}\right)^2$

$I_{core} = I_{double-cone}$

$I_{double-cone} = \frac{3}{10}M_{dc}\left(R_{dc}\right)^2 + \frac{1}{5}M_{dc}h_{dc}^2$

Figure 4: The comparison of moment of inertia formulae for each core shape: sphere rotating about its center and double-cone rotating end over end.

Again, using a physical ratio of the rotational inertia formula for each object we get the following mathematical expression;

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} \approx \frac{\left(\frac{3}{10}M_{double-cone}\left(R_{double-cone}\right)^2 + \frac{1}{5}M_{double-cone}\left(h_{double-cone}\right)^2\right)}{\left(\frac{1}{4}M_{cylinder}\left(R_{cylinder}\right)^2 + \frac{1}{12}M_{cylinder}\left(L_{cylinder}\right)^2\right)}$$

Assumption #1 **–** cores of constant density, equal mass, and similar dimension (where the radius and height of the cylinder is equal to the radius and height of the double cone), then;

$$M_{double-cone} = M_{cylinder} = M$$

and

$$R_{double-cone} = R_{cylinder} = h_{double-cone} = L_{cylinder} = R$$

we get the following result

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} \approx \frac{\left(\frac{3}{10}M\left(R\right)^2 + \frac{1}{5}MR^2\right)}{\left(\frac{1}{4}M\left(R\right)^2 + \frac{1}{12}MR^2\right)} \cong \frac{3/10 + 1/5}{1/4 + 1/12} \cong \frac{1/2}{1/3} \cong 1.50$$

Theoretically, once the double-cone core is revolving with a given spin speed it will resist changes to that motion more than the solid cylinder core. If we picked a rotational axis somewhere between the two extremes of the double-cone core shape (base-axis or vertex-to-vertex axis) the moment of inertia (ratios) values will be somewhere between the two results (1.20-1.50) that have been obtained. Therefore, it is safe to say that the solid double-cone core shape will have more rotational inertia than the solid cylinder shape of the same constant density, mass, and relative size.

Theoretical Comparisons

Rotational Energy Considerations: Cylinder vs. Diamond Core

What about rotational energy? Using ***Assumption #2*** that each core is set revolving about is rotational axis to the same rotational velocity we will highlight the differences of rotational kinetic energy.

Comparing the double-cone shape with the vertex to vertex axis to a cylinder rotating around any principle axis.

$$K.E._{rotational} = \frac{1}{2} I\omega^2$$

$$\frac{\text{Diamond Core } KE_{rotational}}{\text{Solid Cylinder Core } KE_{rotational}} = \frac{\frac{1}{2}\left(I_{diamond}\right)\omega_{diamond}{}^2}{\frac{1}{2}\left(I_{cylinder}\right)\omega_{cylinder}{}^2}$$

$$I_{diamond} = \frac{3}{5} M_{diamond} R^2_{diamond}$$

$$I_{cylinder} = \frac{1}{2} M_{cylinder} R^2_{cylinder}$$

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} = \frac{\frac{1}{2}\left(\frac{3}{5} M_{diamond} R^2_{diamond}\right)\omega_{diamond}{}^2}{\frac{1}{2}\left(\frac{1}{2} M_{cylinder} R^2_{cylinder}\right)\omega_{cylinder}{}^2}$$

Highlighting the "shape" differences in the rotational energy with both objects spinning with the same rotational velocity (ω), both with the same overall mass (M), and, finally, both cores same dimension (R) the preceding mathematical ration reduces to a result we have already seen.

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} = \frac{\frac{1}{2}\left(\frac{3}{5} MR^2\right)\omega^2}{\frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2} = \frac{\frac{1}{2}\left(\frac{3}{5}\right)}{\frac{1}{2}\left(\frac{1}{2}\right)} = \frac{6}{5} \cong 1.2$$

Comparing the double-cone shape with the base-diameter-axis to a cylinder rotating around any central-diameter-axis using the same equivalent measures, we can quickly get to;

$$I_{diamond} = \frac{3}{10} M_{dc} \left(R_{dc}\right)^2 + \frac{1}{5} M_{dc} \left(h_{dc}\right)^2$$

$$I_{cylinder} = \frac{1}{4} M_{cylinder} \left(R_{cylinder}\right)^2 + \frac{1}{12} M_{cylinder} \left(L_{cylinder}\right)^2$$

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} M_{dc}\left(R_{dc}\right)^2 + \frac{1}{5} M_{dc}\left(h_{dc}\right)^2\right) \omega_{diamond}{}^2}{\frac{1}{2}\left(\frac{1}{4} M_{cylinder}\left(R_{cylinder}\right)^2 + \frac{1}{12} M_{cylinder}\left(L_{cylinder}\right)^2\right) \omega_{cylinder}{}^2}$$

$$\frac{\text{Diamond Core}}{\text{Solid Cylinder Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} MR^2 + \frac{1}{5} MR^2\right)}{\frac{1}{2}\left(\frac{1}{4} MR^2 + \frac{1}{12} MR^2\right)} = \frac{\frac{1}{2}\left(3/10 + 1/5\right)}{\frac{1}{2}\left(1/4 + 1/12\right)} \cong 1.50$$

the same result as was found earlier when just the moment of inertia was compared.

When comparing the moment of inertia of the core shapes we can't help but to also compare rotational kinetic energy differences the core shapes.

We have discovered that a diamond shaped core will have more rotational kinetic energy than a solid cylinder shaped core under nearly equivalent measurement and rotational conditions.

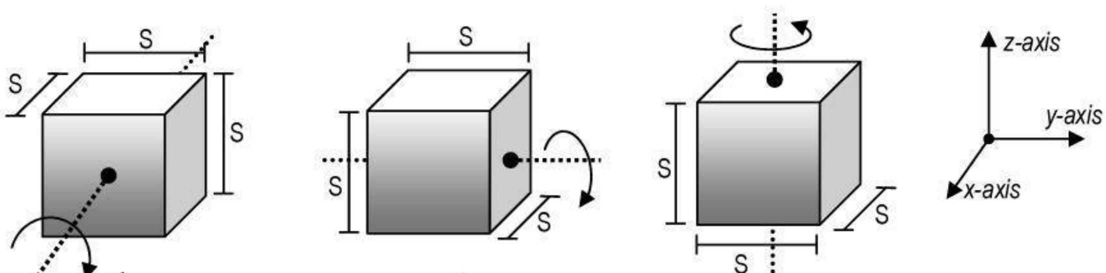
THEORETICAL COMPARISONS: MOMENT OF INERTIA

Rotational Inertia Considerations: Cube vs. Diamond Core

Basic Assumptions for Comparisons:

1. Each core is of similar dimension, constant density, and will have the same mass. (equal size, weight, and smooth mass distribution)
2. Each core is set revolving about is rotational axis to the same rotational velocity. (equal spin speed)
3. Each core rotates about an axial geometric center. (axis through center)

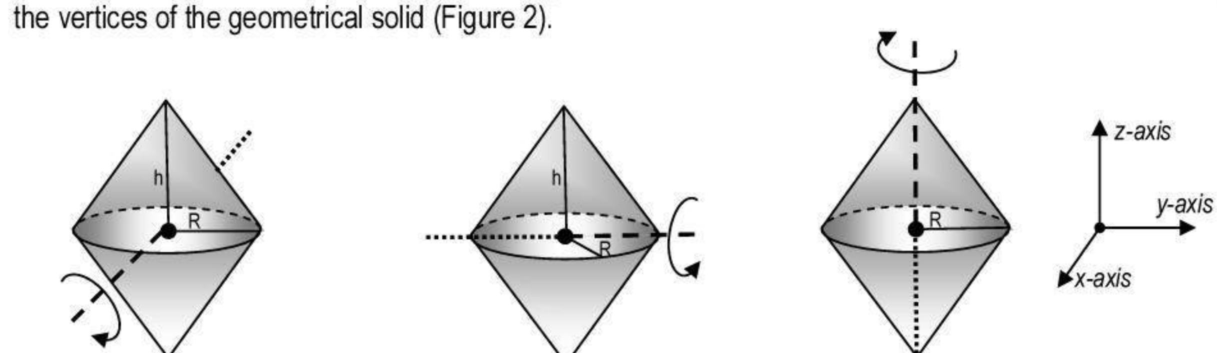
The calculated theoretical moment of inertia for a solid cubical mass (I) was determined to be the same for any given axis through its geometrical center (due to the high degree of symmetry—Figure 1). In fact, a cube will have the same moment of inertia with any axis through its center.



$$I_{x-axis} = \frac{1}{6}MS^2 \qquad I_{y-axis} = \frac{1}{6}MS^2 \qquad I_{z-axis} = \frac{1}{6}MS^2$$

Figure 1: The moment of inertia (I) formulae around each principle axis for a solid cube.

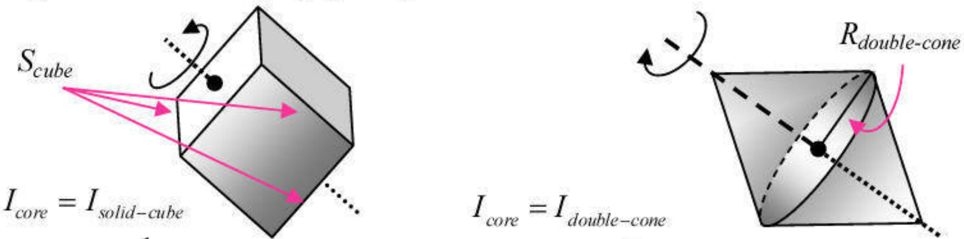
The calculated theoretical moment of inertia for a double solid cone (I) was determined to be the same for the axes through the base of each cone while different for an axis through the vertices of the geometrical solid (Figure 2).



$$I_{x-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2 \qquad I_{y-axis} = \frac{3}{10}MR^2 + \frac{1}{5}Mh^2 \qquad I_{z-axis} = \frac{3}{5}MR^2$$

Figure 2: The moment of inertia (I) formulae around each principle axis for a double solid cone (approximate diamond core shape).

Now a comparison: If we compare a "diamond core" (double-right cone) to a solid cube core around the following central rotational axes we can see that the rotational inertia will be different for each core since the calculated moment of inertia formula differs due to the different mass distribution about the rotational axis for each geometric solid shape (Figure 3).



$$I_{core} = I_{solid-cube}$$
$$I_{solid-cube} = \frac{1}{6} M_{cube} \left(S_{cube}\right)^2$$

$$I_{core} = I_{double-cone}$$
$$I_{double-cone} = \frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2$$

Figure 3: The comparison of moment of inertia formulae for each core shape.

Using ***Assumption #1*** described earlier that each core is of constant density, both cores have the same mass, and each is of similar dimension (where the side length of the cube core is equal to the radius of double cone), then;

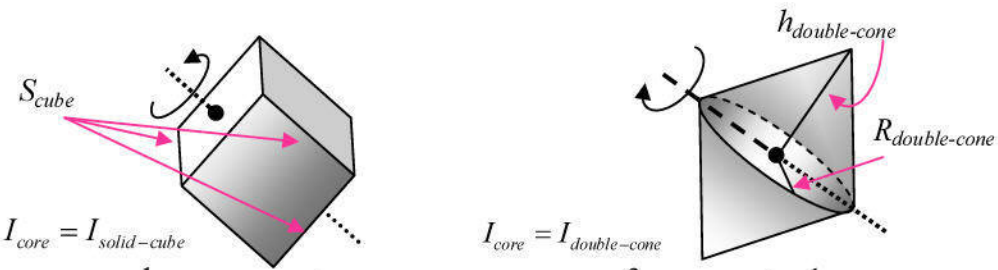
$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} \approx \frac{\left(\frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2\right)}{\left(\frac{1}{6} M_{cube} \left(S_{cube}\right)^2\right)}$$

$$M_{double-cone} = M_{cube} = M$$
and
$$R_{double-cone} = S_{cube} = R$$

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} \approx \frac{\left(\frac{3}{5} M_{double-cone} \left(R_{double-cone}\right)^2\right)}{\left(\frac{1}{6} M_{cube} \left(R_{cube}\right)^2\right)} \cong \frac{\left(\frac{3}{5} MR^2\right)}{\left(\frac{1}{6} MR^2\right)} \cong \frac{3/5}{1/6} \cong 3.6$$

All we have done is compare the moment of inertia of two cores where the only principle difference between them is the mass distribution about the rotational axis. Once the double-cone core is revolving with a given angular velocity (spin speed) it will resist changes to that angular velocity **much** more than the cubical core.

Now a different comparison: Now we compare a "diamond core" (double-right cone) to a cubical core around the following rotational axes. (This would have the diamond core rotating vertex over vertex.)



$$I_{core} = I_{solid-cube} \qquad I_{core} = I_{double-cone}$$

$$I_{solid-cube} = \frac{1}{6} M_{cube} \left(S_{cube}\right)^2 \qquad I_{double-cone} = \frac{3}{10} M_{dc} \left(R_{dc}\right)^2 + \frac{1}{5} M_{dc} h_{dc}^{\;2}$$

Figure 4: The comparison of moment of inertia formulae for each core shape: cube rotating about its center and double-cone rotating end over end.

Again, using a physical ratio of the rotational inertia formula for each object we get the following mathematical expression;

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} \approx \frac{\left(\frac{3}{10} M_{double-cone} \left(R_{double-cone}\right)^2 + \frac{1}{5} M_{double-cone} \left(h_{double-cone}\right)^2\right)}{\left(\frac{1}{6} M_{cube} \left(S_{cube}\right)^2\right)}$$

Assumption #1 – cores of constant density, equal mass, and similar dimension (where the side length of the cube is equal to the radius and height of the double cone), then;

$$M_{double-cone} = M_{cube} = M$$

and

$$R_{double-cone} = S_{cube} = h_{double-cone} = R$$

we get the following result

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} \approx \frac{\left(\frac{3}{10} M \left(R\right)^2 + \frac{1}{5} MR^2\right)}{\left(\frac{1}{6} M \left(R\right)^2\right)} \cong \frac{3/10 + 1/5}{1/6} \cong \frac{1/2}{1/6} \cong \; 3.0$$

Theoretically, once the double-cone core is revolving with a given spin speed it will resist changes to that motion much more than the solid cube. If we picked a rotational axis somewhere between the two extremes of the double-cone core shape (base-axis or vertex-to-vertex axis) the moment of inertia (ratios) values will be somewhere between the two results (3.0-3.6) that have been obtained. Therefore, it is very safe to say that the solid double-cone core shape will have more rotational inertia than the solid cube shape of the same constant density, mass, and relative size.

Theoretical Comparisons

Rotational Energy Considerations: Cube vs. Diamond Core

What about rotational energy? Using ***Assumption #2*** that each core is set revolving about is rotational axis to the same rotational velocity we will highlight the differences of rotational kinetic energy.

Comparing the double-cone shape with the vertex to vertex axis to a cube rotating around any principle axis.

$$K.E._{rotational} = \frac{1}{2} I\omega^2$$

$$\frac{\text{Diamond Core } KE_{rotational}}{\text{Solid Cube Core } KE_{rotational}} = \frac{\frac{1}{2}\left(I_{diamond}\right)\omega_{diamond}{}^2}{\frac{1}{2}\left(I_{cube}\right)\omega_{sphere}{}^2}$$

$$I_{diamond} = \frac{3}{5} M_{diamond} R^2_{diamond}$$

$$I_{cube} = \frac{1}{6} M_{cube} S^2_{cube}$$

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{5} M_{diamond} R^2_{diamond}\right)\omega_{diamond}{}^2}{\frac{1}{2}\left(\frac{1}{6} M_{cube} S^2_{cube}\right)\omega_{sphere}{}^2}$$

Highlighting the "shape" differences in the rotational energy with both objects spinning with the same rotational velocity (ω), both with the same overall mass (M), and, finally, both cores same dimension (R) the preceding mathematical ration reduces to a result we have already seen.

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{5} MR^2\right)\omega^2}{\frac{1}{2}\left(\frac{1}{6} MR^2\right)\omega^2} = \frac{\frac{1}{2}\left(\frac{3}{5}\right)}{\frac{1}{2}\left(\frac{1}{6}\right)} = \frac{18}{5} \cong 3.6$$

Comparing the double-cone shape with the base-diameter-axis to a cube rotating around any central-axis using the same equivalent measures, we can quickly get to;

$$I_{diamond} = \frac{3}{10} M_{dc} (R_{dc})^2 + \frac{1}{5} M_{dc} (h_{dc})^2$$

$$I_{cube} = \frac{1}{6} M_{cube} (S_{cube})^2$$

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} M_{dc} (R_{dc})^2 + \frac{1}{5} M_{dc} (h_{dc})^2\right) \omega_{diamond}^{\ 2}}{\frac{1}{2}\left(\frac{1}{6} M_{cube} (S_{cube})^2\right) \omega_{cube}^{\ 2}}$$

$$\frac{\text{Diamond Core}}{\text{Solid Cube Core}} = \frac{\frac{1}{2}\left(\frac{3}{10} MR^2 + \frac{1}{5} MR^2\right)}{\frac{1}{2}\left(\frac{1}{6} MR^2\right)} = \frac{\frac{1}{2}\left(3/10 + 1/5\right)}{\frac{1}{2}\left(1/6\right)} \cong 3.0$$

the same result as was found earlier when just the moment of inertia was compared.

When comparing the moment of inertia of the core shapes we can't help but to also compare rotational kinetic energy differences the core shapes.

We have discovered that a diamond shaped core will have more rotational kinetic energy than a solid cubical shaped core under nearly equivalent measurement and rotational conditions.